

# A unified management of congestions due to voltage instability and thermal overload

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## Abstract

This paper proposes two approaches for a unified management of congestions due to voltage instability and thermal overload in a deregulated environment. Both techniques aim to remove, in some optimal manner, voltage and thermal congestions stemming from base case or post-contingency states, by a simultaneous handling of operating and security constraints with respect to several contingencies. The objective of the first approach is to adjust the market-based power injections (generator output and possibly load consumption) at the least cost while the second one aims at curtailing power transactions in a transparent and non-discriminatory way. These techniques rely on sensitivities which pinpoint the best remedial actions against congestions owing to voltage instability and thermal overload. Numerical results with both approaches are provided on a realistic 80-bus system model.

*Key words:* voltage stability, thermal overload, dynamic security analysis, congestion management, generation rescheduling, load curtailment

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## 1 Introduction

Nowadays the process of electricity market deregulation has prevailed in many countries. Depending on the particular characteristics of every power system, various forms of electricity services unbundling have been implemented. However, despite specific achievement differences, two conceptual models emerged: the pool model and the bilateral contract model [1–3].

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In a deregulated environment a system is said to be “congested” when some specified operating constraints (e.g., branch current, bus voltage magnitude, etc.) or security constraints (e.g., thermal, voltage stability, angle stability, etc.) are violated in the current or in a foreseen operating state. Operating constraints refer to the normal system configuration (in “N”) while security constraints refer to “N-1” and some plausible “N-k” system configurations. Congestion management consists in controlling the system such that all operational and security constraints are satisfied. Whatever the implemented deregulation model, the Transmission System Operator (TSO) is responsible for relieving or removing congestions in foreseen operating states (established after the day-ahead market clearing) as well as in real-time. Clearly, power systems were confronted to congestions in the vertically integrated environment as well. In this environment congestion management was most often performed by modifying the economic dispatch at the least cost until no operating or security constraint was violated.

*Thermal overload* and *voltage instability*, the main concern in this paper, are two significant causes of congestions in many power systems.

The available means to remove congestions linked to voltage instability, – actions on voltages through transformer ratios, generator voltages and reactive power injections – are somewhat limited either by the range of variation allowed for these variables or by their impact on the pre-contingency system configuration. For instance, in order to restore voltage security, it is unlikely that large amounts of shunt compensation can be switched (if available) in the pre-contingency configuration, owing to the risk of overvoltages. The same holds true for generator terminal voltages. Additionally, the above-mentioned control variables are usually set in the day-ahead market, by running an Optimal Power Flow (OPF) to minimize the transmission losses. On the other hand, active power generation rescheduling and load curtailment can have a significant impact on both voltage stability and thermal overloads. However, these actions have a cost and hence must be taken in a transparent and widely agreed manner. In the sequel we will concentrate on congestions that cannot be removed by “cost-free” means such as: capacitors, transformer taps, phase shifters, FACTS devices, topological changes, etc.

The methods to tackle congestions can be divided into two main categories [4–6]: *economical* (e.g., market splitting, auctioning) and *technical* (e.g., generation redispatch, transaction curtailment). The approaches considered in this work fall in the second category.

This paper proposes two congestion management approaches that differ by the control means. A first approach, referred to as *Injection Control* (IC), relies on power injections, i.e., generator productions and load consumptions. A second approach, referred to as *Transaction Control* (TC), relies on power

transactions. These approaches are suited to deal with congestions appearing in day-ahead market clearing as well as in real time.

The IC approach can be implemented in any deregulated model. It consists in modifying the market-based generation scheme at the least cost, according to the generator bids [1,3,5,7–12]. In order to ensure higher competition this method can be easily extended to load curtailment [9,10,13,14].

As an alternative, the TC approach is applied in deregulated systems operated under the bilateral contract model. It consists in curtailing non-firm transactions in some optimal manner in order to relieve congestions [2,15–18].

The management of *thermal overload congestions* has been widely analyzed [1,2,5,7,11,12,14,16,18]. It is most often based on the very fast DC load flow model. Although valid in many practical cases, the latter approximation may be less satisfactory when voltage and thermal aspects are strongly coupled as well as under highly load conditions.

The management of *voltage instability congestions* has been comparatively less investigated so far [8–10,13,19]. Most of these approaches aim at keeping security margins with respect to plausible contingencies above some threshold. Multiple contingencies are treated through heuristics [8,13] or through constrained optimization [9,10,19].

Up to now both problems have been considered separately because voltage stability analysis requires more accurate tools than a mere DC load flow. This work proposes an integrated handling of both problems.

The benchmark when dealing with voltage instability and thermal overload congestions is a classical security constrained OPF [20] including voltage stability constraints [19,21,22]. This approach has, however, two drawbacks: (very) high dimensionality, especially when many contingencies have to be incorporated, and static modelling of voltage instability phenomena by algebraic load flow equations. Therefore, it may not have the accuracy and robustness of time-domain methods, while being heavy for real-time applications.

We propose instead simplified optimization approaches coupled with the fast time-domain Quasi-Steady-State (QSS) simulation [23,24] used to evaluate the system response to contingencies. The QSS simulation allows a more accurate modelling of the voltage instability phenomena and yields, at very low computational cost, sensitivities indicating the best remedial actions to remove congestions due to voltage instability [25] and thermal overload [26].

As regards the security criteria used for congestion management, it has been argued that the deterministic “N-1” security criterion is too conservative (e.g., [27–30]). In the deregulated context, the “N-1” criterion is felt by some au-

thors as an obstacle to competition. First, it is deemed to yield unnecessarily high operating costs. Second, it does not take into account the likelihood of the various contingencies, but rather treats them all as equiprobable. On the other hand, during severe weather conditions likely to affect transmission lines or in view of the non negligible probability of having protection failures, the “N-1” (or even the “N-2”) criterion may not provide enough security, as demonstrated by some blackouts worldwide (e.g., North America 2003, Italy 2003, etc.). The latter events raised reticences against the “take-risk” congestion management strategies. The future is most probably in a careful tradeoff between preventive congestion management and corrective (emergency) control [27–30]. The objective will be to minimize the overall cost of both preventive and corrective actions. However, while the cost of preventive actions is rather easy to calculate, getting a reliable estimate of the corrective costs is a challenging problem for voltage unstable scenarios as well as for severe post-contingency thermal overloads.

In our congestion management framework, we take into account the traditional requirement that none of the specified contingencies causes voltage instability nor thermal overload, and no branch is overloaded in the base case situation. Any contingency causing voltage instability or thermal overload is labelled harmful. Otherwise it is said harmless.

The paper is organized as follows. Section 2 presents the derivation of voltage and thermal security constraints. The IC and TC approaches are successively presented in Section 3 and Section 4, respectively. Section 5 offers some numerical results with the proposed methods while some conclusions are drawn in Section 6.

## 2 Linearized security constraints

If the power system is deemed voltage and/or thermal insecure, the TSO should modify the pre-contingency operating point in a such a way that voltage and/or thermal security are restored. To this end the TSO needs to know *where* and of *how much* to act in order to optimally remove congestions taking care that these actions do not create other security violations. To tackle this problem one needs to derive security constraints. The latter take on the form of linear inequality constraints and are obtained as explained hereafter. We first derive these constraints for power injections (generator active power and load consumption) as control variables and then extend them to transactions as well.

## 2.1 Voltage security constraints

We proposed in Ref. [25] a sensitivity computation to rank the candidate remedial actions against low or unstable voltages. To this purpose we use the sensitivities of the bus voltage magnitude experiencing the largest drop in the post-contingency voltage unstable scenario with respect to power injections  $\mathbf{P}$ , which we denote  $\partial V_\ell / \partial \mathbf{P}$ . Within the QSS simulation, these sensitivities are computed at each simulation step while, for control ranking purposes, we use the sensitivity values obtained just after the system trajectory crosses the so-called “critical” point (which can be either a saddle-node bifurcation (SNB) or a breaking point (BP)). The latter situation is identified by a sign change of these sensitivities.

We showed that, in voltage unstable cases, the proposed sensitivities, computed in the neighborhood of a SNB or a BP, yield essentially the same bus power ranking as the eigenvector/normal vector computation proposed in previous works [23,31]. We also stressed that the proposed sensitivities exhibit advantages over the eigenvector-based ones in terms of efficiency, reliability and extension to low but stable voltages. It is thus of interest to express the existing normal vector based voltage security constraints by means of  $\partial V_\ell / \partial \mathbf{P}$  sensitivities.

Following the procedure proposed in Refs. [10,23,31] a linear voltage security constraint can be obtained for each voltage unstable post-contingency scenario  $r$  ( $r = 1, \dots, u$ ),  $u$  being the number of unstable contingencies at the base case:

$$\mathbf{n}_r^T \Delta \mathbf{P} \leq \mathbf{n}_r^T (\mathbf{P}_r^c - \mathbf{P}_r^d) \quad (1)$$

where  $\mathbf{P}_r^d$  is the power injection vector at the desired, yet never attained, post-contingency long-term equilibrium<sup>2</sup>,  $\mathbf{P}_r^c$  is the power injection vector at the critical point and  $\mathbf{n}_r$  is the normal vector to the post-contingency voltage stability region at  $\mathbf{P}_r^c$ . This inequality expresses that the variation  $\Delta \mathbf{P}$  of power injections would bring  $\mathbf{P}_r^d$  inside the post-contingency voltage stability region, whose boundary is linearly approximated.

As mentioned above, Ref. [25] shows that, at the critical point of the system, the normal vector  $\mathbf{n}_r$  and the  $\partial V_{\ell r} / \partial \mathbf{P}$  sensitivities are practically collinear, i.e.,

$$\mathbf{n}_r = k_r' \left( \frac{\partial V_{\ell r}}{\partial \mathbf{P}} \right) \quad (2)$$

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<sup>2</sup>  $\mathbf{P}_r^d$  is somewhat different from the base case power injection vector  $\mathbf{P}^0$  due to post-contingency frequency regulation and load sensitivity to voltage.

where  $V_{\ell_r}$  is the voltage which drops the most in the  $r$ -th unstable post-contingency scenario.

The sign of  $k'_r$  can be obtained as follows. Assume that all components of  $\mathbf{P}$  are taken positive when the power enters the network. Suppose furthermore that the sensitivities are computed just *after* crossing the critical point (in a two-bus system, this means evaluating the sensitivities at a point on the lower part of the QV curve, near its nose). Then, at bus  $\ell$ , one has  $\partial V_{\ell_r}/\partial Q_{\ell_r} < 0$  [24]. The component of  $\mathbf{n}_r$  relative to  $Q_{\ell_r}$  is also negative, i.e.,  $n_{Q_{\ell_r}} < 0$ , since the normal vector points towards the outside of the feasible region and restoring an equilibrium point requires to increase the reactive power production at bus  $\ell$ . From these two inequalities and (2) it follows that  $k'_r$  is positive. Introducing (2) into (1) and dividing by  $k'_r$  yields the voltage security constraint:

$$\left(\frac{\partial V_{\ell_r}}{\partial \mathbf{P}}\right)^T \Delta \mathbf{P} \leq \left(\frac{\partial V_{\ell_r}}{\partial \mathbf{P}}\right)^T (\mathbf{P}_r^c - \mathbf{P}_r^d)$$

or in a more practical form:

$$\sum_{i=1}^m \frac{\partial V_{\ell_r}}{\partial P_i} \Delta P_i \leq C_r \quad r = 1, \dots, u \quad (3)$$

where  $m$  is the number of system buses and  $C_r = \left(\frac{\partial V_{\ell_r}}{\partial \mathbf{P}}\right)^T (\mathbf{P}_r^c - \mathbf{P}_r^d)$  is a known constant for the  $r$ -th contingency.

Remarks:

1. Clearly, if the sensitivities were computed just before crossing the critical point,  $k'_r$  would be negative and the inequality (3) should be reversed.
2. The left hand side of (3) can be interpreted as the opposite of the linearized expected change of the most affected voltage.
3. The  $\partial V_{\ell_r}/\partial \mathbf{P}$  sensitivities are computed in the post-contingency system configuration and used in the pre-contingency one. The information provided by these sensitivities was validated on several power system models [25,26]. Obviously, the limit of validity of these sensitivities is that of linearization. Their main purpose is to determine the *relative efficiency* of the various candidate actions. Nonlinear simulations are needed to determine the optimal *amount* of control action as will be explained in Section 3.3.

## 2.2 Thermal security constraints

Thermal security constraints express that no branch current is above its limit after any specified contingency as well as in the base case:

$$I_{jr} \leq I_j^{max} \quad r = 0, \dots, c \quad j = 1, \dots, b \quad (4)$$

where  $b$  is the number of branches,  $c$  the number of specified contingencies,  $I_{jr}$  the current in the  $j$ -th branch after the  $r$ -th contingency ( $r = 0$  refers to the pre-contingency base case situation) and  $I_j^{max}$  is the maximum current allowed in the  $j$ -th branch.

It is well known that (pre- or post-contingency) branch currents vary rather linearly with bus power injections. The inequality (4) can thus be linearized into:

$$\sum_{i=1}^m \frac{\partial I_{jr}^0}{\partial P_i} \Delta P_i \leq I_j^{max} - I_{jr}^0 \quad (5)$$

where  $I_{jr}^0$  is the post-contingency branch current for the base case value of the injections, and the partial derivative denotes the sensitivity of post-contingency branch current to pre-contingency injection. The latter can be determined using the DC load flow approximation [5] or from a well-known sensitivity formula involving the Jacobian of the steady-state equations (standard AC load flow [32] or long-term equilibrium equations [26]).

## 2.3 Extension of security constraints to transactions

Under the bilateral contract model, it is of interest to quantify security with respect to transactions. To this purpose, voltage and thermal security constraints should be derived considering transactions as control variables. A simple, linear change of variables can be used, as detailed hereafter.

A transaction is a bilateral exchange of power between a selling and a buying entity. In the sequel, the selling (resp. buying) entity is called *source* (resp. *sink*) and may comprise several generators (resp. loads). The  $k$ -th transaction ( $k = 1, \dots, t$ ) is defined by its volume  $T_k$ , which is the active power received by the sink, as well as by the bus participations in the source and the sink. The latter are defined by the two  $m$ -dimensional vectors:

$$\boldsymbol{\alpha}_k = [\alpha_{k1} \dots \alpha_{ki} \dots \alpha_{km}]^T \quad \boldsymbol{\beta}_k = [\beta_{k1} \dots \beta_{ki} \dots \beta_{km}]^T$$

where  $\alpha_{ki}$  (resp.  $\beta_{ki}$ ) is the participation factor of the generator (resp. load) at bus  $i$  in the  $k$ -th transaction. Obviously,  $\alpha_{ki} = 0$  (resp.  $\beta_{ki} = 0$ ) in the absence of a participating generator (resp. load) at bus  $i$ , and  $\alpha_{ki} > 0$  (resp.  $\beta_{ki} > 0$ ) otherwise. Furthermore, the participation factors are chosen such that:

$$\sum_{i=1}^m \alpha_{ki} = 1 + \delta_k \quad \sum_{i=1}^m \beta_{ki} = 1$$

where  $\delta_k$  accounts in an approximate way for the transmission losses associated with the  $k$ -th transaction.

Thus, for the  $k$ -th transaction, the active power  $P_{ki}^+$  ( $i = 1, \dots, m$ ) *injected into* and the active power  $P_{ki}^-$  *drawn from* the  $i$ -th bus relate to the above variables through:

$$P_{ki}^+ = \alpha_{ki} T_k \quad P_{ki}^- = \beta_{ki} T_k \quad P_{ki}^+, P_{ki}^- \geq 0$$

and the volume of the  $k$ -th transaction is given by:

$$T_k = \sum_{i=1}^m P_{ki}^- = \frac{\sum_{i=1}^m P_{ki}^+}{1 + \delta_k} \quad (6)$$

Denoting by  $\Delta$  the variations from base case values, we have:

$$\Delta P_{ki}^+ = \alpha_{ki} \Delta T_k \quad \Delta P_{ki}^- = \beta_{ki} \Delta T_k$$

and, for all transactions, the net power variation at bus  $i$  is:

$$\Delta P_i = \sum_{k=1}^t \Delta P_{ki}^+ - \Delta P_{ki}^- = \sum_{k=1}^t (\alpha_{ki} - \beta_{ki}) \Delta T_k \quad (7)$$

This equation defines a mapping between the power injection and the transaction spaces.

Since a transaction is nothing but a linear combination of power injections, security constraints can be derived with respect to transactions as a direct extension of those derived with respect to power injections.

Taking (7) into account, the voltage security constraints (3) can be rewritten in terms of transactions as:

$$\sum_{k=1}^t \frac{\partial V_{\ell_r}}{\partial T_k} \Delta T_k \leq C_r \quad (8)$$



where  $\frac{\partial V_{\ell r}}{\partial T_k} = \sum_{i=1}^m \frac{\partial V_{\ell r}}{\partial P_i} (\alpha_{ki} - \beta_{ki})$ .

Analogously, the thermal security constraints (4) become

$$I_{jr} + \sum_{k=1}^t \frac{\partial I_{jr}}{\partial T_k} \Delta T_k \leq I_j^{max} \quad (9)$$

where  $\frac{\partial I_{jr}}{\partial T_k} = \sum_{i=1}^m \frac{\partial I_{jr}}{\partial P_i} (\alpha_{ki} - \beta_{ki})$  represents the sensitivity of current  $I_{jr}$  to a change in transaction  $T_k$ .

### 3 Injection control approach

#### 3.1 Optimization formulation

Let the base case be characterized by the injections vector  $\mathbf{P}^0$ . We decompose each power injection correction into  $\Delta P_i = \Delta P_i^+ - \Delta P_i^-$  with  $\Delta P_i^+, \Delta P_i^- \geq 0$ .

The TSO objective is very often to remove a congestion at the least cost:

$$\min_{\Delta P_i^+, \Delta P_i^-} \sum_{i=1}^m (c_i^+ \Delta P_i^+ - c_i^- \Delta P_i^-) \quad (10)$$

where, for a generator which can be rescheduled,  $c_i^+$  (resp.  $c_i^-$ ) is the incremental (resp. decremental) bidding price, while for a load which can be curtailed,  $\Delta P_i^+ = 0$  and  $c_i^-$  is the curtailment price. The cost of congestion is further allocated to the market actors as an uplift cost [1,3,8,33].

Under the linear voltage and thermal security constraints derived in Section 2, the congestion management can be obtained as the solution of the following Optimization Problem (OP):

$$\min_{\Delta P_i^+, \Delta P_i^-} \sum_{i=1}^m (c_i^+ \Delta P_i^+ - c_i^- \Delta P_i^-) \quad (11)$$

$$\text{s.t. } \sum_{i=1}^m \frac{\partial V_{\ell r}}{\partial P_i} (\Delta P_i^+ - \Delta P_i^-) \leq C_r \quad (12)$$

$$\sum_{i=1}^m \frac{\partial I_{jr}}{\partial P_i} (\Delta P_i^+ - \Delta P_i^-) \leq I_j^{max} - I_{jr}^0 \quad (13)$$

$$\sum_{i=1}^m (\Delta P_i^+ - \Delta P_i^-) = 0 \quad (14)$$

$$0 \leq \Delta P_i^+ \leq P_i^{max} - P_i^0 \quad (15)$$

$$0 \leq \Delta P_i^- \leq P_i^0 - P_i^{min} \quad (16)$$

The voltage security constraints (12) may be written for any harmful contingency  $r$  ( $r = 1, \dots, u$ ) at  $\mathbf{P}^0$ . The thermal security constraints (13) may be written for each branch  $j$  ( $j = 1, \dots, b$ ) in each post-contingency state ( $s = 1, \dots, c$ ) as well as in the base case ( $s = 0$ ) which leads to  $(c + 1) \times b$  constraints. Nevertheless, in order to keep the problem tractable and because most thermal security constraints are not limiting, we derive them only for the branches close or above their limits in the post-contingency states. The inequalities (15, 16) correspond to limits on either the generated power or the maximum load power that can be curtailed. Finally, Eq. (14) is the overall power balance, assuming that losses will not change significantly. If this is not deemed acceptable, a full (security constrained) OPF incorporating (12, 13, 15 and 16) can be used.

Note that, in the above formulation, controls are of active power nature, but reactive aspects can be taken into account as well in the computation of the voltage security sensitivities as explained in [10].

The relationships (11-16) make up a standard linear programming problem. Let  $\Delta \mathbf{P}^*$  be its solution. Since (13) but even more (12) are only linear approximations, the “corrected” operating point  $\mathbf{P}^* = \mathbf{P}^0 + \Delta \mathbf{P}^*$  may be still (hopefully slightly) voltage and/or thermal insecure, or conservatively secure. Moreover, one cannot exclude the case where a contingency would create both voltage and thermal problems: a contingency which triggers voltage instability at  $\mathbf{P}^0$  is considered harmful from the voltage viewpoint but, as the system does not reach an operating point where branch overloads can be checked, a possible thermal problem is hidden.

We propose a two-step procedure to deal with such situations:

- (1) *voltage security restoration*. One first ensures that no contingency causes voltage instability any longer. To this purpose, the voltage security constrained optimization problem (11, 12, 14, 15, 16) is solved;
- (2) *thermal security restoration*. When all contingencies are voltage stabilized, thermal overloads are checked; if any branch is overloaded, the corresponding constraint (13) is added and a new optimization is performed, in order to eliminate overloads while maintaining voltage security.

A flow chart of this approach is presented in Fig. 1, where  $\Delta \mathbf{P}_V^*$  is the solution of step (1).

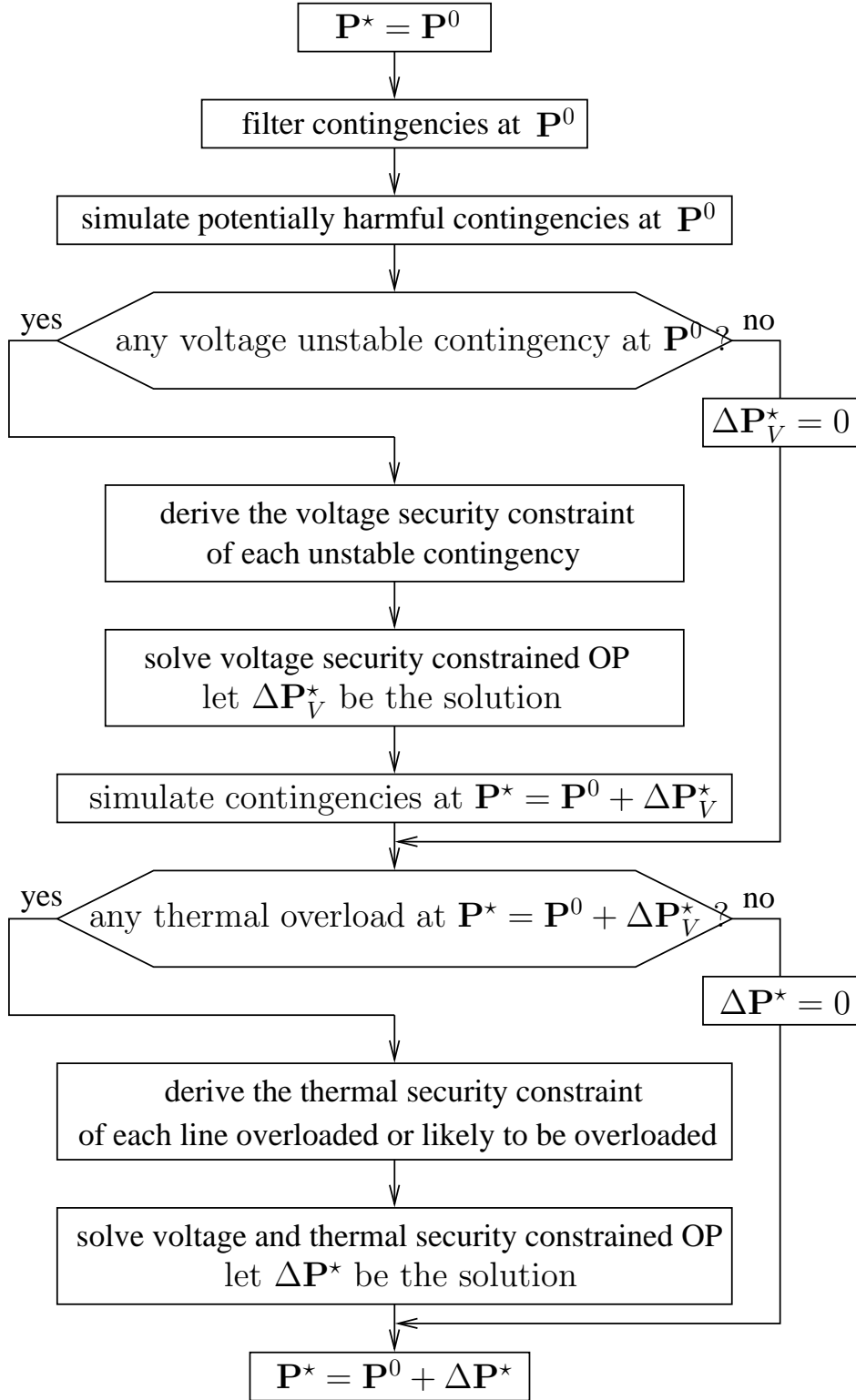


Fig. 1. IC approach algorithm

Alternatively, one can first “partially” restore voltage and thermal security by solving the problem (11-16) except for the thermal constraints corresponding

to voltage unstable scenarios. The latter constraints are checked at the solution and, if any of them is violated, it is added to the previous set and the so enlarged problem is solved.

Note that after the voltage security restoration phase, new contingencies may become unstable at the new operating point  $\mathbf{P}^0 + \Delta\mathbf{P}_V^*$ . For each such contingency, a new voltage security constraint is derived and added to the already existing set, and the so enlarged optimization problem is solved. The procedure can be repeated until all contingencies are voltage stable at  $\mathbf{P}^0 + \Delta\mathbf{P}_V^*$ . The same procedure can be performed after thermal security correction.

Note finally that the base case injections vector  $\mathbf{P}^0$  can be further adjusted (e.g., with some load and/or generation increase) such that, by running the congestion management procedure, to restore in fact a desired security margin with respect to any contingency [10].

### 3.2 Contingency filtering

In real-time applications, it is essential to quickly filter out harmless contingencies and limit the analysis to the (potentially or effectively) dangerous ones.

As regards voltage instability, we use a procedure similar to the one described in [34]. The various contingencies are simulated in the base case  $\mathbf{P}^0$  with an AC load flow and only those leading to divergence or causing voltage drops larger than some threshold are labelled potentially harmful. The latter are analyzed in greater detail by QSS simulation, which filters out the false alarms. The remaining, harmful contingencies are incorporated into the congestion management procedure.

As regards thermal overloads, each post-contingency operating point provided by the AC load flow calculation (at the first filtering step) or by a stable QSS simulation (at the second step) is checked with respect to branch overloads. The corresponding contingencies are also included in the congestion management procedure.

### 3.3 Heuristic handling of nonlinearities

We present now a heuristic technique to handle the nonlinearity of voltage and thermal security constraints.

For the  $r$ -th unstable contingency, we consider that the *relative* efficiency of

the various controls is known, while the *amount* of control may not be known accurately, due to nonlinear effects. In other words, we assume in (12) that the *relative* values of the various  $\partial V_{\ell r} / \partial P_i$  sensitivities are correct while  $C_r$  may be affected by some error. The latter comes in particular from the linear approximation of underlying Eq. (3). To obtain a correct value for this constant, we replace (12) by:

$$\sum_{i=1}^m \frac{\partial V_{\ell r}}{\partial P_i} (\Delta P_i^+ - \Delta P_i^-) \leq f_r C_r \quad (17)$$

and we solve the optimization problem (11, 17, 14, 15, 16) adjusting  $f_r$  iteratively to obtain the best objective function together with a voltage secure point  $\mathbf{P}^*$ . The bisection method is used to this purpose; it consists in building a smaller and smaller interval  $[f_u, f_a]$ , such that the solution  $\Delta \mathbf{P}^*$  of the linearized optimization problem (11, 17, 14, 15, 16) yields a voltage secure point  $\mathbf{P}^*$  for  $f_r = f_a$  and an insecure one for  $f_r = f_u$ . This is checked through the QSS simulation of the  $r$ -th harmful contingency. The procedure is repeated until the absolute difference between two successive objectives (11) becomes smaller than a tolerance, in which case  $f_r$  is set to  $f_a$ . The search starts with  $f_a = 1$ ,  $f_u = 0.2$  if the very first optimization yields a secure operating point  $\mathbf{P}^*$ , and with  $f_a = 5$ ,  $f_u = 1$  if it yields an insecure one. Note that the value of  $f_r$  can be also seen as a measure of how much the values of sensitivities are over- or underestimated with respect to their real values. The above initialization ( $f_u = 0.2$  and  $f_a = 5$ ) means thus that we assume the real values of sensitivities are not more than 5 times over- or underestimated.

This technique is applied to each constraint (12) (i.e., to each unstable contingency) separately. As a by-product, we obtain the control change required to make the system secure with respect to each contingency separately.

In principle, the same iterative procedure can be also used to find more accurate thermal constraints. However, a simpler technique exploiting the more linear nature of this problem can be used instead. Thus, once the post-contingency current  $I_{jr}^{real}$  has been obtained by QSS simulation, all sensitivities  $\frac{\partial I_{jr}}{\partial P_i}$  are multiplied by:

$$\frac{I_{jr}^{real} - I_{jr}^0}{\sum_{i=1}^m \frac{\partial I_{jr}}{\partial P_i} \Delta P_i^*} \quad (18)$$

where the numerator is the real change in branch current between the optimum and the base case, and the denominator is the corresponding linear prediction. A single update of the sensitivities is usually enough.

## 4 Transaction control approach

In a liberalized electricity market under the bilateral contract model, simultaneous transactions are submitted to the TSO who is responsible for curtailing some of them if deemed necessary. This implies in turn a modification of power injections at both the sending and receiving buses of the transaction.

Transaction curtailment must be performed in an optimal and transparent manner. Many objectives can be thought of, ranging from the least overall trade curtailment given by the  $L_1$  norm [17] to the (weighted)  $L_2$  norm [16] or the Transmission Loading Relief formula [15].

In this paper we use an  $L_2$ -norm objective, as originally proposed in [16], which consists in minimizing the sum of squared transaction deviations (from the base case values):

$$\sum_{k=1}^t \Delta T_k^2$$

This objective yields a compromise between market forces and system capability. All trades are weighted in terms of MW instead of money, which is non-discriminatory.

The formulation of the TC approach is thus:

$$\min_{\Delta T_k} \sum_{k=1}^t \Delta T_k^2 \tag{19}$$

$$\text{s.t.} \sum_{k=1}^t \frac{\partial V_{\ell r}}{\partial T_k} \Delta T_k \leq C_r \tag{20}$$

$$\sum_{k=1}^t \frac{\partial I_{jr}}{\partial T_k} \Delta T_k \leq I_j^{max} - I_{jr}^0 \tag{21}$$

$$-T_k \leq \Delta T_k \leq 0 \tag{22}$$

As in the IC formulation, voltage security constraints (20) may be written for any contingency  $r$  ( $r = 1, \dots, u$ ) unstable at  $\mathbf{T}^0$ , while thermal security constraints (13) may be written for each branch  $j$  ( $j = 1, \dots, b$ ) in each post-contingency state ( $s = 1, \dots, c$ ) as well as in the base case ( $s = 0$ ). Note that an explicit power balance equation (14) is not required in this formulation, since each transaction (6) is balanced by itself.

The solution  $\Delta \mathbf{T}^*$  of this quadratic programming problem provides the closest distance of the initial set of transactions  $\mathbf{T}^0$  to the secure region defined by

inequalities (20) and (21).

The algorithm of Fig. 1 can be also used in this case, provided that  $\mathbf{P}^0$  is replaced by  $\mathbf{T}^0$ .

Note finally that a mechanism must be found to serve the loads that can not be fully accommodated through bilateral transactions. This situation can be handled by applying successively both TC and IC approaches. Thus, one first finds out the amount of each transaction that can be safely accommodated by solving (19-22). Then, if some transaction curtailment is required, the IC optimization problem (11-16) is solved, where each  $\Delta P_i^-$ 's is the amount of curtailed transaction and only the  $\Delta P_i^+$ 's have to be determined.

## 5 Numerical results

We consider the 80-bus system shown in Fig. 2, a variant of the “Nordic 32” system [35]. A rather heavy power transfer takes place from “North” to “South” areas.

The QSS long-term simulation reproduces the dynamics of load tap changers and overexcitation limiters. Note that there is no slack-bus in the QSS model; instead, generators respond to a disturbance according to governor effects [24]. In this system, the generators of the North area are the only ones to participate in frequency control (i.e., the others have infinite speed droops).

For the sake of comparing the efficiency (in terms of rescheduled MW) of the IC and TC approaches, we consider the particular IC objective corresponding to  $c_i^+ = c^+, i = 1, \dots, m$  and  $c_i^- = c^-, i = 1, \dots, m$ . Taking (14) into account, the objective (10) can be rewritten successively as:

$$\min \sum_{i=1}^m (c_i^+ \Delta P_i^+ - c_i^- \Delta P_i^-) \Leftrightarrow \quad (23)$$

$$\min (c^+ \sum_{i=1}^m \Delta P_i^+ - c^- \sum_{i=1}^m \Delta P_i^-) \Leftrightarrow \quad (24)$$

$$\min (c^+ - c^-) \sum_{i=1}^m \Delta P_i^- \Leftrightarrow \quad (25)$$

$$\min \sum_{i=1}^m \Delta P_i^- = \min \sum_{i=1}^m \Delta P_i^+ \quad (26)$$

The so obtained IC objective will be referred to in the sequel as minimum control rescheduling.

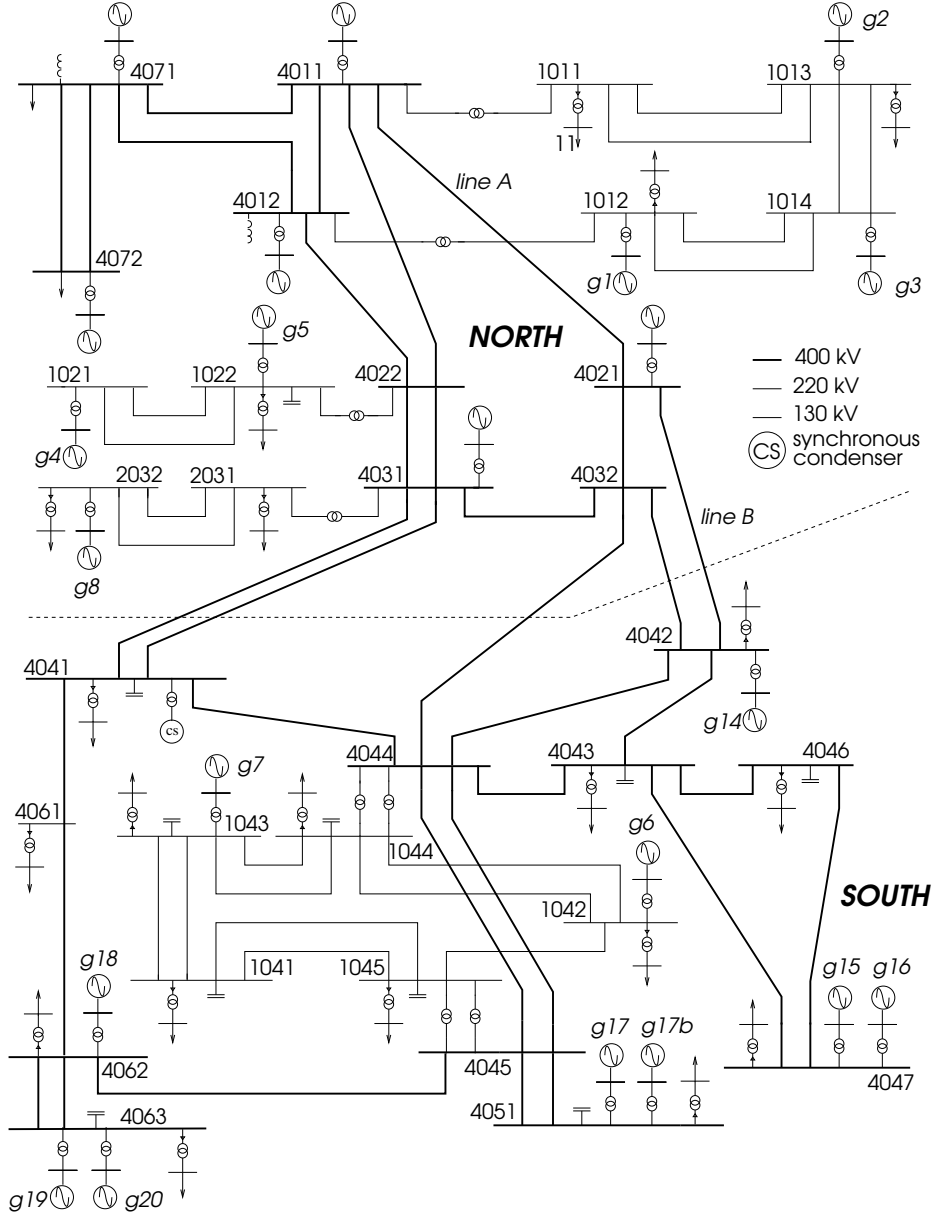


Fig. 2. The (slightly modified) “Nordic 32” test system

We analyze voltage and thermal security with respect to a set of 49 contingencies. At the first step of the procedure (see Fig. 1) 37 harmless contingencies are filtered out using the method described in Section 3.2. The remaining 12 potentially harmful contingencies are analyzed in greater detail by QSS simulation. Among them, 4 false alarms are discarded, the corresponding contingencies being voltage stable. The 41 thrown-out contingencies cause no thermal overload and no branch is likely to be overloaded as a result of a possible voltage security restoration. Most of the 8 harmful contingencies are outages of southern generators, as can be seen from the first column of Table 1. Indeed, voltage security is strongly linked to the power transfer from “North” to “South” areas (see Fig. 2). Since only Northern generators



participate in frequency control, the active power lost in the South part adds to this transfer and prompts instability.

### 5.1 Congestion management through IC approach

We consider hereafter two combinations of controls, whose results are detailed in Table 2. In this table, column V (resp. V+T) provides the voltage (resp. the combined voltage and thermal) congestion correction.

Table 1  
control of contingencies through individual changes

contingency : loss of	generation changes (MW)						objective (MW)
	g4	g5	g6	g7	g16	g17b	
line A		-41		41			41
g8	-42			42			42
g14	-81		39	42			81
g15	-45			42	3		45
g16	-30			30			30
g17	-90			42		48	90
g17b	-25			25			25
g18	-7			7			7

Table 2  
changes in generation or load (MW)

generator or load	case			
	G		GL	
	V	V+T	V	V+T
g3		-66		-75
g4	-90	-90	-78	-78
g7	42	42		
g14		66		
g17b	48	48		
1045			-78	-78
4043				-75
objective (MW)	90	156	78	153

We present hereafter two examples, referred to as “G” and “GL” in the above tables.

#### 5.1.1 Case G: generation rescheduling

For each of the 8 harmful contingencies, the voltage security constraint is identified iteratively, as described in Section 3.3. On the average, this procedure requires 6 iterations (and hence 6 post-control QSS simulations) to meet a 2 MW tolerance (the difference between the objective functions obtained for the marginally stable and unstable values of the multiplier  $f_r$ , respectively). The generation shift needed to restore voltage security is shown in Table 1 for each harmful contingency analyzed separately. As already mentioned, any decrease in generation in the North area, covered by an increase in generation in South area, diminishes the North-South power transfer, and hence enhances voltage security. In this respect the southern generator g7 appears as the “panacea” against all harmful contingencies. On the other hand, among the northern generators, g4 is the one with the greatest benefit for the voltage security.

Coming back to Table 2, one can see that the combination of controls that stabilizes the most dangerous contingency (loss of g17) also stabilizes the other harmful contingencies. Note, however, that stabilizing the worst contingency does not always lead to the stabilization of all harmful contingencies because “conflicting” controls may exist from one contingency to another, as illustrated in [10].

The optimal solution to remove voltage congestion consists of increasing the production of g7 and g17b by 42 MW and 48 MW, respectively, and decreasing the one of g4 by 90 MW. The so obtained voltage secure operating point is next checked with respect to thermal overloads. It is found that the loss of line A would cause the current in line 4031-4032 to reach 107 % of its admissible value, while the outage of one circuit of line 4022-4031 would bring the other circuit at 87 % of its limit. The thermal constraints relative to these two branches are thus incorporated to the optimization problem. As can be seen from the table, the solution includes the voltage congestion correction, together with an additional shift of 66 MW between g14 and g3 to remove the thermal overload. In this simple example, the overall optimal solution is the sum of the voltage and thermal corrections.

#### 5.1.2 Case GL: generation rescheduling and load curtailment

Next, we provide an example where both generation rescheduling and load curtailment are allowed to remove a voltage instability congestion. The maximum interruptible fraction of each load has been limited to 20 % and the

power factor is assumed to remain unchanged after shedding. As can be seen from Table 2, the obtained correction consists of shedding 78 MW (and the corresponding 28 Mvar) at bus 1045, located in the voltage sensitive area, and compensating on the remote generator g4. With respect to Case G, the objective function (11) reaches a lower value (78 MW) thanks to the larger number of controls offered.

## 5.2 Congestion management through TC approach

We now consider transactions as control variables. For easy comparisons, we consider the same (voltage and thermal) congested operating point as in Section 5.1 but we assume now that this base case situation would stem from the ten transactions detailed in Table 3.

Table 3

Description of the requested transactions

transaction	source(s)	sink(s)	$T_d(MW)$
$T_1$	g21	g18	40
$T_2$	g1, g2, g3	g15, g16	50
$T_3$	g4, g5	g17, g17b	40
$T_4$	g9, g10	1041, 1045	50
$T_5$	g1, g2, g3	1011, 1012, 1013, 1014	30
$T_6$	g8	2031	10
$T_7$	g19, g20	4045	20
$T_8$	g12	4044	10
$T_9$	g14	g8	20
$T_{10}$	g18	g11	20

For comparison purposes, we first use an  $L_1$  norm ( $\sum_{k=1}^t \Delta T_k$ ) to remove the congestion due to voltage instability. Column A in Table 4 shows that the solution of the corresponding optimization problem consists in merely reducing transactions  $T_2$  and  $T_4$ , that have the greatest impact on voltage security. Adding thermal security constraints to the optimization problem leads to curtail two more transactions ( $T_1$ , and  $T_3$ ) as shown in column B of the same table.

The  $L_1$  norm is “unfair” because it leads to curtailing transactions by decreasing order of their impact on security. In the above example, solving the voltage and thermal congestions leads to removing the whole transactions  $T_4$ ,  $T_3$  and  $T_2$  from the market.

Expectedly, this effect is attenuated when using the  $L_2$  norm, as shown by columns C and D in Table 4, which correspond to columns A and B, respectively. The quadratic objective leads to cutting down some more power (-178.3 MW vs. -161.1 MW for the  $L_1$  norm) but the effort is distributed over the transactions in a fairer way.

Table 4

Curtailment of transactions for voltage and thermal security restoration

transaction	A	B	C	D
$\Delta T_1$		-21.1	-24.4	-38.2
$\Delta T_2$	-43.5	-50	-25.3	-41.4
$\Delta T_3$		-40	-15.4	-30.8
$\Delta T_4$	-50	-50	-26.8	-42.8
$\Delta T_5$			-0.1	-0.2
$\Delta T_6$			-5.2	-7.5
$\Delta T_7$			-2.9	-5.3
$\Delta T_8$			-10	-10
$\Delta T_9$				
$\Delta T_{10}$				-2.1
$\sum_k \Delta T_k$	-93.5	-161.1	-110.1	-178.3

One can observe that acting on transactions instead of power injections is less efficient. For instance, when acting on power injections to restore voltage security (with norm  $L_1$ ), one needs to either curtail 78 MW of load or reschedule 90 MW of generation (see Table 2), while 93.5 MW of transactions have to be curtailed. The same applies for the  $L_2$  norm: the 80 MW load curtailment and 94 MW generation rescheduling are smaller than the 110.1 MW transaction curtailment. Using power transactions as control variables is less efficient because to the fact that each transaction is a linear combination of power injections which may contain less efficient injections.

## 6 Conclusion

This paper proposes two optimization-based approaches (IC and TC) for a unified management of congestions due to voltage instability and thermal overload. They are well suited to the day-ahead and real-time environments. The IC (resp. TC) approach takes on the form of a linear (resp. quadratic) programming problem, that can be easily handled by standard solvers. The core of these techniques is the computation of sensitivities that rank the candidate

remedial actions. Both approaches allow the simultaneous treatment of all harmful (and possibly some harmless) contingencies. Heuristic techniques for handling the nonlinearities of voltage and thermal security constraints have been also proposed.

The IC and TC approaches apply whether the market is based on the pool or the bilateral model. They could be straightforwardly extended to the hybrid pool-bilateral model adopted in an increasing number of countries to improve competition.

A natural extension of the proposed techniques is to consider start-up (and possibly shut-down) costs of generators, which would lead to integer programming problems [11].

Finally, the TC approach can be easily applied to the cross-border capacity allocation by coordinated auctioning, a system used by several TSOs in Europe [4]. A weighted  $L_1$  norm is to be used to this purpose.

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